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ABSTRACT

The use of writing-to-learn in mathematics classes is one way teachers can implement both communication and problem-solving goals. This study investigates the effects of implementing an integrated, experimenter-designed writing program within an existing basic text of Algebra II. The program consisted of specifically formulated lessons in writing designed to enhance the students' understanding of topics studied. The study used two sections of the class as an experimental group (N=34) and two as the control group (N=34) with sections randomly assigned to treatment groups. Writing activities, both transactional and expressive, were integrated within the experimental group's lessons. Data collection instruments included tests, writing attitude scales, mathematics attitude scales, and student writing. Results indicate that there were no significant differences between the experimental and control group in the tests or attitudes, neither were there any significant differences between the pre- and post-test attitudes of writing or mathematics for either group. Trends in the data suggest that a longer study might have resulted in the experimental group performing significantly higher than the control group on an achievement measure. Also included is a summary of personal research and personal reflections. Contains 22 references. (JRH)

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EFFECTS OF USING WRITING-TO-LEARN MATHEMATICS

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EFFECTS OF USING WRITING-TO-LEARN MATHEMATICS

Rebecca Finley Kasparek

INTRODUCTION

Schoenfeld (1992) describes the purpose of mathematics education as learning to think mathematically. To think mathematically "means (a) developing a mathematical point of view - valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing the competence with the tools of the trade, and using those tools in the understanding structure - mathematical sense-making" (p. 335). In 1989, the National Council of Teachers of Mathematics (NCTM) placed communication as a high priority for grades K-12 stating that the "mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can: reflect upon and clarify their thinking about mathematical ideas and relationships; and express mathematical ideas orally and in writing" (NCTM, 1989, p. 140). Recognizing a related concern, in 1980 NCTM recommended that "problem solving be the focus of school mathematics in the 1980's" (NCTM, 1980, p. 2). Then in 1989, NCTM strongly endorsed the 1980 recommendation. "The development of each student's ability to solve problems is essential if he or she is to become a productive citizen" (NCTM, 1989, p. 6).

The use of writing-to-learn in mathematics classes is one way teachers can implement both communication and problem solving goals. Emig (1977), Vygotsky (1962), Odell (1980), and Imscher (1979) all emphasize the important link between writing and learning. They believe that writing in a content area can encourage students to analyze, compare facts, and synthesize material. "Writing helps tie down ideas and make connections between old and new concepts" (Vygotsky, 1962, p. 92).

Many mathematics educators agree that writing should become a part of the daily routine of every mathematics class. Johnson (1983) suggests that, if students can write clearly about mathematical concepts, they can then probably understand them. Bell and Bell (1985) state that writing has been an effective and practical tool for teaching mathematics problem solving. Asking students to write about a process or a problem requires them to clarify their thoughts. This writing procedure then becomes an integral part of the thought process. Writing forces students to become active rather than passive learners, and thus they are more likely to be actively involved in "constructing" their own knowledge.

This study investigated the effects of implementing an integrated, experimenter-designed writing program within an existing basic text of Algebra II. This program consisted of specifically formulated lessons in writing designed to enhance the students' understanding of topics studied.

SUMMARY OF RESEARCH ON WRITING-TO-LEARN MATHEMATICS

Some of the more important findings regarding writing-to-learn mathematics will be summarized briefly. It will be recalled that, with learning

to think mathematically as a primary purpose of mathematics education, increased emphasis has been placed in recent years on having the mathematics curriculum include the development of language so that students can communicate mathematical ideas. In the forefront of research in this area has been that on writing-to-learn mathematics, which is an outgrowth of the movement for writing-in-content.

Writing-in-content is writing to learn. Educators believe such writing encourages learning and thinking. They further claim that writing-in-content helps students develop the abilities to analyze and synthesize. In light of these beliefs, pertinent research has been done on types of writing, student-teacher interaction, and language and concept development.

Many types of writing activities, ranging from journals to free writing to term papers have been the subjects of research. While many students' writing activities in mathematics are limited to notetaking, others have been found to profit markedly from both transactional writing and expressive writing. Researchers have observed that the writing process changed the students from passive learners to active thinker-participants. It was also observed that students had better understanding and retention when they wrote about mathematical processes and problems.

Through regular reading of student writings, teachers were able to better observe and remedy problems and misconceptions in the learning process, a benefit which the students also appreciated. However, it was found that, for writing to be effective, it has to be integrated into the mathematics class and the teacher has to be involved in the process and responsive to the students.

Research on language and concept development has shown that language has a strong influence on concept development. For instance, when students generate their own language and ideas, comprehension is increased. Research has also demonstrated that mathematics language skills, combined with procedural skills, are related to conceptual understanding. Researchers in these areas consistently recommend incorporating mathematical language activities into current teaching practices.

According to Resnick (1987), research has shown that learning does not happen by passive absorption alone; students approach learning with a background of knowledge, take in the new information, and construct their own meanings. Instructional methods, such as writing-to-learn, have the potential to make passive students into active students, actively constructing their own knowledge (Kenyon, 1988). Through the writing process, students gather and organize old and new knowledge and synthesize it into their own structure of knowledge (Nahrgang & Peterson, 1986). "As students write down, reflect on, and react to their thoughts and ideas, they enhance their executive problem-solving skills" (Kenyon, 1988, p. 8). Thus, self-monitoring or metacognition skills are improved.

Schoenfeld (1980) and Silver, Branca, and Adams (1980) suggest that instruction that emphasizes the metacognitive aspects of problem solving can be effective in helping students develop awarenesses of their own thought processes. There is strong evidence that, by making students aware of their own thought processes, learning can be enhanced. Silver (1982) and Schoenfeld (1983) have found that helping problem solvers reflect on their cognitive processes also helps to bring the students to an awareness of their emotional reactions to problem solving. Writing as an instructional tool has the potential to achieve that.

According to Connolly (1989), writing-to-learn is reflective and questioning. "The basic purpose is to help students become independent, active learners by creating for themselves the language essential to their

personal understanding" (p. 6). Writing activities can be used to involve the students actively in analytical thinking and reflecting on their own learning (Miller & England, 1989). By writing and reflecting on their own mathematical experiences, the students become active learners. Thus, writing can have a significant impact on learners' cognition and metacognition (Powell & Lopez, 1989). One of the most important concepts for mathematics education has been expressed in the following statement: "Now we are beginning to realize that writing is not just the end product of learning; it is a process by which learning takes place" (Griffin, 1983, p. 121).

Bell and Bell (1985) recommend that all mathematics teachers use writing as a part of the daily routine. Hirsch and King (1983) recommend integrating writing into the mathematics class. Ganguli (1989), in research with college algebra students, found that an integrated writing program was more effective than the traditional program. He suggests that the integration of writing into mathematics instruction deserves much more attention, and that studies which examine the effects of such programs are needed.

The research that has been reviewed is consistently enthusiastic about the potential value of writing to learn mathematics. Existing formal research and informal studies done by teachers from elementary school to college consistently point toward both the advantages of using writing in the mathematics class and the need for further research in this area.

SUMMARY OF THE STUDY

This study investigated the effects of implementing an integrated, experimenter-designed writing program within an existing basic text for Algebra II. The study took place at a private school in the southeastern part of the United States during the 1992-93 school year. The experimenter taught the four sections of Algebra II used for this study. Enrollment in each section was approximately 15-20 students. For this study, two sections were used as the experimental group and two as the control group. Sections were randomly assigned to treatment groups. Due to normal attrition, a total of 68 students, 34 in the experimental group and 34 in the control group, participated in the study.

Both the control and experimental group received the same instruction using a basic Algebra II text. Writing activities, both transactional and expressive, were integrated within the experimental group's lessons. Data for the study were collected in several ways. At the beginning of the study, each student was given a preliminary algebra test, a writing attitude scale, and a mathematics attitude scale. During the study, students completed the appropriate chapter tests. After completing study of the first two of four chapters, students were given the midtest. At the conclusion of the study, students were given the posttest, the writing attitude scale, and the mathematics attitude scale. Following each of the chapter tests, the midtest, and the posttest, students were asked to explain in writing how they worked two preselected items. These explanations were scored holistically.

The primary hypothesis was that students receiving the integrated writing program in Algebra II within the framework of the basic text would exhibit greater achievement than those students receiving the regular instruction in the basic text. The secondary hypothesis was that those students receiving the integrated writing program would develop more positive attitudes towards mathematics and writing. All hypotheses were tested at the .05 level of significance.

For the mathematics achievement test data and the writing sample data, analysis of covariance (ANCOVA) was used to test the differences between the groups. The scores on the achievement tests and writing samples were used as the dependent variables and the pretest was used as the covariate.

The results for the algebra performance tests were mixed. For the midtest, posttest, Chapter 2 test, and Chapter 4 test, no significant difference was found between the experimental group and the control group. On the Chapter 3, Chapter 5, and Chapter Test average, the experimental group performed significantly higher than the control group.

ANCOVA was also used to analyze the data on the writing samples. Again, the results were mixed. No significant difference was found between the experimental and control group on midtest, posttest, or combined mid and posttest. For the individual chapter tests, the combined chapter tests, and the total writing samples the experimental group performed significantly higher than the control group.

The attitude scales were analyzed in several ways. Paired samples t-tests were used to compare each group's pre and post attitudes in both writing and mathematics. Independent samples
t-tests were used to compare the pre and post attitudes between groups for both writing and mathematics. No significant differences were found between the pre and post attitudes of writing or mathematics for either group. No significant differences were found between the experimental and control groups.

CONCLUSIONS

Although there was insufficient evidence to support the hypothesis, the results of the midtest and posttest show a trend. The difference between the means of the experimental group and the control group grew from .17 on the pretest, to .91 on the midtest, to 1.30 on the posttest. The pretest, midtest, and posttest each had a maximum score of 30. The test statistic was 1.639 for the midtest and 3.216 for the posttest, which approached the critical value of 4.0. This trend in the data suggests that a longer study might have resulted in the experimental group performing significantly higher than the control group on an achievement measure.

The chapter test data also demonstrated a distinctive trend; the differences between the means of the experimental group and control group were 4.88 on the Chapter 2 test, 8.47 on the Chapter 3 test, and 7.71 on the Chapter 5 test. The maximum score on the chapter tests was 100. The test statistic was 3.188 for Chapter 2, 5.586 for Chapter 3, and 9.568 for Chapter 5. The data, both from the algebra tests and the writing samples, suggest that the writing activities had a cumulative positive effect on the mathematical achievement of the experimental group and that with a study of longer duration, the hypothesis might have been supported. However, one cannot discount that many factors may have been involved in this widening of differences between the groups. Since the students were aware of the purpose of the study, for example, the experimental groups' achievement may have been influenced by the Hawthorne effect.

The smaller difference in the means for the Chapter 4 test, 2.77, was probably the result of a chapter test that was too easy for all the classes, and thus there was a ceiling effect. Chapter 4 was on matrices, which all the students enjoyed and on which they felt very much at ease with their understandings of the concepts. The means for both groups on this test were substantially higher than on other tests.

For the writing sample data, the differences between the means of the experimental and control groups also demonstrated a trend. On the midtest the difference was 0.38, and on the posttest the difference was 0.83. In the chapter tests, the differences between the means were 1.32 on Chapter 2 writing samples, 2.50 on Chapter 3 writing samples, 1.76 on Chapter 4 writing samples, and 2.26 on Chapter 5 writing samples. The test statistic for the writing samples also increased from 0.543 on the midtest to 1.903 on the posttest. On the chapter tests writing samples, the test statistic increased from 6.587 on Chapter 2, to 11.481 on Chapter 3, to 11.880 on Chapter 4 , and finally to 14.161 on Chapter 5. This trend suggests a potential cumulative effect for the writing samples.

It is possible that the writing-to-learn activities were more effective on the chapter tests than on the mid and posttests because the material covered by the chapter tests more closely correlated with the material the students were currently studying and writing about. In addition, the free-response type format of the chapter tests, as opposed to the multiple-choice type format of the midtest and posttest, required the students to work out the problems and show their work, which forced them to be more mindful of their procedures. By comparison, the midtest and posttest, which had been constructed by selecting items primarily from ERB Algebra tests and supplemented by multiple-choice items from the basic text's evaluation manual, resembled standardized achievement tests. These multiple-choice items were written with traditional instruction in mind, and might have been biased toward the control group who had not been concentrating on explaining their procedures.

These differences in the types of tests may also be related to the differences in the data collected from the writing samples. In the standardized-type format of multiple-choice items, the students were not forced to describe the process they used to arrive at an answer and at times resorted to guessing. In such cases, they may have had a more difficult time later in explaining their answers. Another factor involved in the difference between the results of the writing samples from the midtest/posttest and chapter tests may have been that the writings the students had been involved in for each chapter were so recent that they may have felt more secure with expressing their thoughts. By comparison, the writing sample items on the midtest/posttest may have been less recent and the students may have been less sure of their explanations. The writing samples for the midtest were especially low because the second problem they had to respond to was a linear-programming problem at the end of the test. Many of the students did not get that far on the test and since the problem was more difficult than many others, some opted not to do it.

In the analysis of the attitude scales, no significant differences were found between the experimental group and the control group. In addition, there was no significant differences found between the pre attitude and post attitude of either group. A brief background of these classes may help to explain these results and the stability of these measurements. Since the experimenter had taught the majority of the students the year before either in Honors or Regular Geometry, the classroom routine was not changed substantially. Furthermore, the majority of the students were well motivated and were from homes in which education is valued highly. The students were used to a demanding curriculum requiring a large amount of writing.

SUMMARY OF PERSONAL RESEARCH

From conducting and being a part of this study, it has become clear to the experimenter that the use of writing in the mathematics class has many advantages both for the students and the teacher. A primary advantage is communication--communication about learning and communication about feelings. The second standard advocated by the NCTM is that students should learn "to communicate mathematical ideas so that all students can: reflect on and clarify their thinking about mathematical ideas and relationships" (NCTM, 1989, p. 140). By having students explain their understandings, insights were obtained into the ways they were learning or not learning and teaching strategies could be developed or altered. By having students confide their concerns, insights could be obtained into their feelings and these could be addressed.

The benefits to the students were many. As the students were writing, they were actively involved in constructing their own knowledge and making it more personal. They were able to organize and to reflect on what they were learning. For many, this organization and reflection increased their knowledge of mathematics. Many students in both the experimental and control group realized as they wrote their explanation to a writing sample item that they had made a mistake when initially working the problem. By taking the time to write about the problem, the students understood the problem better.

The writing also benefited the students therapeutically as they expressed their feelings, attitudes, and concerns. The students felt that the teacher cared about them individually and was concerned about their needs as a person as well as their needs regarding mathematics.

The benefits to the teacher were also numerous. By reading the students' writings, the teacher gained insights into the students' understandings and misunderstandings. Immediate feedback could be given to the classes in the form of reteaching or reexplanations, paying special attention to topics that had been misunderstood. The teacher was better able to evaluate the students' progress by reading their writings. By only looking at a quiz or a test, the teacher had not been able to evaluate as effectively what the student did or did not understand.

By reading the students' expressions of concern, the teacher was better able to address the needs of the students. If the students were concerned about the pace of the class, frustrated with their understanding of a certain concept, or annoyed by a person sitting next to them, their concerns were made known through their writings and thus could be addressed.

Based on the data collected from this study, it seems that the students in the experimental group performed somewhat better than the control group on the chapter tests and the writing samples from the chapter tests. Furthermore, the data suggest that when students are tested on the material they are taught and tested in a similar format, they improve by using writing as a tool for learning. The trend in the data suggests that the experimental group might have performed significantly higher than the control group on the posttest had the experiment lasted longer. From the data collected, it would appear obvious that the use of writing-to-learn mathematics is potentially a valuable tool for both the students and teachers and that it should not be overlooked by the mathematics teacher.

PERSONAL REFLECTIONS

The students in all classes were excited to be included in this study and all cooperated willingly. Because the experimenter had taught most of the students the previous year, there was an environment of trust already established, so the students were comfortable expressing their concerns and questions to the teacher. In the experimenter's judgment, in this atmosphere of open expression, an important additional advantage of using writing in the mathematics class was the regular feedback it gave to the teacher from all of the students. The teacher had the opportunity daily to have students' reaction to the teaching procedures, so there was constant adjustment to the needs of the students. Thus, misconceptions and confusions that were obvious from reading the students' writings could be used as a signal to modify instruction for all the students. For example, early in the study, it was obvious from their writings that at least two students were confused about reversing inequality signs when solving inequalities. The students thought that, if there were a negative sign anywhere in the inequality, the inequality had to be reversed. Had it not been for the regular writing activities, this confusion might not have been noted for some time. Another misconception that was discovered from the writings was a confusion about slope. One student thought that the slope of a line either perpendicular or parallel to a given line was the negative reciprocal. Another student thought that, since parallel lines were inconsistent, two lines had to be perpendicular to be classified as independent and consistent. One other student confused slope and equation, stating, "The equation of a vertical or horizontal line is undefined and 0."

Closely related to the advantages of feedback was the improved communication between student and teacher which resulted from the use of writing in the mathematics class. This was especially beneficial in this study for the quiet student and, more particularly, for the quiet struggling student. One such student, in reflecting back about the writings she had done, said:

> The writing exercises did help me because they help me communicate with you and it let you know how I felt about the material. When I had to write down what I learned it helped to reinforce the material and my understanding of it.

From reading the writings of students, the experimenter gained insights into their difficulties. For example, there were several struggling students whose difficulties were far better (or perhaps, for the first time) understood from reading their writings than had been possible in working with them in individual oral sessions or in interviews during the previous year. From grading these students' quizzes and tests, it would have appeared that they knew very little of the mathematics. However, from reading their writings, it was obvious they knew much more about the subject than showed on tests. It was usually minor flaws in their reasoning or simple misconceptions that were causing them trouble. One such student wrote the following summarizing the chapter on matrices:

> I learned that a matrix is a system of rows and columns. It varies in dimensions. It can be 3 X 2, 2 X 2, 3 X 3, 1 X 3, etc. I also learned that two matrices are equal only if they have the same dimensions and their corresponding elements are equal. I

learned how to multiply scalar multiplication and to add matrices.

The above dialogue, together with correct illustrations of all the procedures, continued as the student summarized the rest of the chapter. One other student who is not always as successful on tests as he would like to be wrote the following about linear equations:

> We have come to find that when given two points, you can substitute into the formula $\frac{y_2 - y_1}{x_2 - x_1}$ the coordinates for the points. Lines that are parallel are going to have the same slope, but perpendicular lines have the negative reciprocal slope. We can determine the equation of a line when given one point and the slope by putting the given numbers into the $y = mx + b$ formula.

The following examples of the students' reflections on the mathematical content they were studying demonstrated, in part, their understandings of the concepts.

> A linear equation is an equation whose graph is a line. A linear equation is identifiable by its containing one or two variables with no variable having an exponent other than one.

> To find the slope, you take the difference in y over the difference in x. That equals the slope. Then, to find the y-intercept, you take a set of points and the slope and put them into the equation $y = mx + b$, then figure out "b" which represents the y-intercept. Once you have done all that, just place the values into the equations; $y = mx + b$ and $Ax + By = C$.

> In graphing a line, solve the equation for y. So for example, if you have the equation $3y = -2x - 6$, divide the equation by 3. Now you have $y = -(2/3)x - 2$. Now to graph it, you would start at -2 on the y-axis and go over 3 and down 2 and connect the points.

> To find out if the lines are perpendicular, parallel, or neither, you only need the slope. If the first slope is 3/5 and the second slope is - 5/3, so the product is -1, so they are perpendicular.

> Some special functions are direct variation, constant, and identity functions. A linear function in the form of $y = mx + b$ where $b = 0$ and m is not = 0, is called a direct variation. If $b = 0$ and $m = 1$, then it is an identity function. If $m = 0$, it is a constant function.

> Lines that intersect have different slopes and are consistent and independent. There is only one solution for lines that intersect. Lines that coincide have the same slope and the same intercepts. These lines are consistent and dependent and have infinite solutions. Lines that are parallel have the same slope and different intercepts. They are inconsistent and have no solution.

> In this chapter, we have learned several ways to solve systems of equations, some hard and some easy. We learned to solve by

graphing. If you graph the two equations, then you could find where they intersect. For solving systems of equations by substitution, you would solve one of the equations for one of the variables, then you could solve the rest by plain algebra. For elimination, you get one of the variables to cancel out, then you can solve for the other. Next you just substitute to find the first variable.

To solve three equations with three variables, I took two of the equations and eliminated the z, then I chose one of the equations I had already used and used the one I had not used. I eliminated the z again and had two equations for x and y. I then eliminated again for x and solved for y. I then substituted for y and got what x equaled. I then substituted for x and y in the original equation and solved for z.

To solve a linear programming problem, you must first define the variables. You must incorporate these into inequalities. You would then be able to graph these and then you should be able to see the feasibility region. It will look like some sort of polygon. You will then need to record the coordinates of the vertices that make this polygon. You will then need to determine the maximum or minimum expression. Then substitute the coordinates into the expression and find the answer.

When you have two matrices and you need to combine them through multiplication, first you have to make sure the matrices can be multiplied. The first matrix (A) must have the same number of columns as the second matrix (B) has rows. Next you multiply the first row of A by each column of B. Then you add each product of the first number of the first column with the second number of the second column. Now you do the same as with the first row, but with the second row. After all the steps are completed you should have a new matrix.

To evaluate the three by three determinant, I used the diagonal method and copied the first two columns of the matrix to the right side. I drew diagonal lines going from top to bottom, multiplied each row, then added them together. I then drew diagonal lines going from bottom to top, multiplied them together, and then subtracted them.

To solve a system of equations as a matrix equation, the first thing I did was write the system as a matrix equation. Then I found the inverse of the first part and multiplied both sides of the equation by it. I checked my answer by substitution to make sure it was right.

To multiply three binomials together, you take the first two factors and multiply. What you get from that, you multiply by the next factor. Then you combine alike terms.

The first five of these quotes of the students' reflections on the mathematical content demonstrate that the students had a sufficient basic

understanding of the concepts to define and identify a linear equation, to find the slope and the equation of a line given two points, to graph a line when in slope-intercept form, to determine if two line are parallel, perpendicular, or neither, and to identify some special linear functions.

The next three quotes pertaining to systems of linear equations demonstrated that the students also had a good understanding of the terms inconsistent, consistent and dependent, and consistent and independent, and of the processes used to solve systems of two and three equations. The quote on linear programming demonstrated that the student was aware of the process and complex procedures necessary to solve problems of this type.

The three quotes on matrices and determinants demonstrated the students' understandings of the complex processes used in some matrix operations and in evaluating three-by-three determinants. The final quote, which explains the process used to multiply three binomials together, shows a good understanding of this process.

In writings such as these, the students are forced to use mathematical terminology and to be precise with their explanations. By taking a process and explaining it in their own words, the students make the particular process a part of their own knowledge and skills. Furthemore, reading writings such as these gave the experimenter one additional way to evaluate the students' understanding. Studying the students' explanations as well as their algebraic and numerical work gave the experimenter a much clearer picture of the concepts the students were, or were not, understanding.

The following quotes are the reflections of two students on the relationship between two chapters studied.

> I feel that the relationship between chapters 2 and 3 is very evident and needed. Without the information and tactics that are learned in chapter 2, there wouldn't be anyway that someone could fully understand chapter 3. Concepts like the coordinate system, linear equations, slopes and intercepts are vital to the understanding of chapter 3. An example of this would be the following. To be able to do Linear Programming, you must be able to use many things learned in chapter 2.

> Chapter 2 is a section that's purpose is not to show us ways to solve problems; rather its purpose is to show us techniques that can be used to solve equations. This chapter is like giving us the raw materials to make up an engine, but not showing us how to put it together. Chapter 3 does not serve the same purpose. Chapter 3 explicitly teaches us how to solve problems. With the "raw materials" we learned in Chapter 2, we are now able to put it all together to solve problems.

The preceding two quotes were examples of the type of increase in metacognitive skills that the practice of writing in the mathematics class encouraged. Both of these quotes were from boys who have matured greatly as students this year as they have taken increased responsibility for their own learning.

Several students who had worked very hard in mathematics, both in geometry last year and in algebra this year, without being as successful as they would have liked, were convinced that the writing activities helped them. Two of these students were, by nature, very quiet, so that their writings were

especially helpful to the teacher. One of these, who otherwise might never have had the willingness to so respond, said:

> I think I have done well, considering that I am not very good in math. I feel that all of the writing we did helped me and by my explaining the problem, it gave me more of an understanding for it. Then again, writing is just a technique I learn by well.

Another student stated:

> The writing has helped me a great deal in understanding the problems.

A third, an energetic student who is very good at asking questions in class and at extra-help sessions and is determined to master the material, said:

> The writing our class did during the 4 chapters in the book, in my point of view, helped me extremely. The writing helped me to understand the material too.

One other category of the writing activities that the experimenter found useful was that in which the students were to express their feelings or thoughts about certain topics. This personal written communication between the teacher and student was very beneficial to both. Insights into the concerns the students were having provided the teacher with valuable information to use in addressing all the classes. The following responses were typical answers to a question asking about concerns or problems with the current chapter:

> This chapter (Matrices) has been relatively easy for me until the inverses came into play. I definitely need extra help on that.

> I don't like graphing especially when there are 4 or 5 lines to graph. It gets confusing. I think Cramer's Rule will be helpful later, but it takes too much time.

> This has been the best chapter so far for me. Not only is it the easiest, but I understand it, and it's fun.

> I like this chapter in the sense of it is pretty neat. I do not like how you have to know more than one way to do things. The last couple of chapters have slammed me because they were impossible.

> I think this chapter is very hard. We need to spend a lot of time on this chapter and take it slow.

> I think this chapter is fun and Cramer's Rule is interesting.

> This chapter has its ups and downs. I like everything except for the evaluation of matrices. I am scared for the test.

It was obvious to the experimenter that as the students were progressing, they were becoming more aware of their particular learning style. The students wrote the following about their studying for a test:

> I felt that I studied well and knew the material thoroughly but failed to apply myself on the test.
>
> I was not happy at all with the way I studied for the test. I would have done more problems and looked over the sections I had difficulty with, if I could do it again.
>
> I think my strategy was good and effective. But if I were to change it I would spend more time on understanding the individual sections.
>
> I felt I studied effectively, but I waited to the last minute to start studying.
>
> I reviewed problems with a friend the day before. I would study 3 days before and go to the Math Center on problems I needed help with.
>
> I studied for the last test, both as we went along and the night before. As we covered the sections, I read the information before doing the problems. Then the night before I went over the chapter test. I feel that my strategies were good, but as a way to help myself, I think I could do more problems when I don't understand one of that type.
>
> For the last test, my buddy and I got together to study. We went over the review, one section at a time. If there was something we did not understand, we would stay working on it or ask for help. The next morning, we went to the Math Center and cleared up everything we couldn't clear up ourselves. This was an effective way to study for the test.
>
> I thought my strategies were good but I didn't do well on the test so obviously they weren't. I have no idea what else to do. Before the test I know everything, but during the test my mind gets blank and I get nervous.

In considering the advantages the writing program had, it would be a mistake not to mention two individual students who were directly and positively affected by the writing. One of these students had been having difficulty with test taking in most of his classes for several years. His teachers, advisor, and parents had all been concerned. During discussions in his classes, both mathematics and others, it was obvious that he was comfortable with and knowledgeable about the material. Then he would take the tests and not do well. From reading his writings, it was obvious to this researcher and then to others that he had a very strong understanding of the concepts. In fact, his writings showed a depth of understanding equal to or even surpassing the strongest students in the class. With this additional insight into this student, his advisor and teachers were better equipped to work on the problems of how to assist him in his test taking skills.

The other student that should be mentioned is a particularly shy young man, who had not said a word out loud in mathematics class for the past year and a half. The classroom environment has always been relaxed and non-threatening, and the students have been encouraged to ask questions and not to feel that any question is too trivial to ask. In other words, the classroom has not been a formal one in which a student would fear making a mistake or asking a "dumb question." This very quiet student wrote some of the best, well-organized, and most insightful papers that were received. By reading his writings, the teacher was finally able to realize what a talented student he was. For a year and a half, the teacher had been trying to break through to this young man, to communicate to him and to get him to communicate in return. Since the writing exercises, this student has ventured out of his quietness to come on several occasions to the Math Center to ask the teacher questions of concern to himself, out loud, although very quietly.

Several weeks following the close of the study, as the students were writing comments on their first semester in Algebra II, they were all asked to reflect on the writing they had done and to express their thoughts thereon. All but 3 out of the 34 stated confidently that the writings helped them. Some typical comments were:

> I feel that writing did help me learn Algebra better. When I wrote about a certain topic in math, I found out if I understood it or not.
>
> I feel that the writing helped me organize what we were learning.
>
> The writing assignments keep the ideas going in my head. It also allows me to review the chapter before the test.
>
> I think the writing assignments helped alot. It's been said that writing stuff down helps you alot, so if you have to remember a process, writing it down in the assignments will help.
>
> The writing assignments helped me to see what exactly I didn't know or wasn't sure of.
>
> Since we stopped writing regularly, it has been hard for me to organize my thoughts.
>
> Most of my high test scores came when we were in the experiment. Writing definitely helped me to organize my thoughts.
>
> The writing helped me and when we decreased the writing, I stopped doing as well on tests and quizzes. Please continue writing.

REFERENCES

Bell, E. S., & Bell, R. N. (1985). Writing and mathematical problem solving: Arguments in favor of synthesis. School Science and Mathematics, 85(3), 210-221.

Connolly, P. (1989). Writing and the ecology of learning. In P. Connolly & T. Vilardi (Eds.), Writing to learn mathematics and science (pp. 1-14). New York: Teachers College Press.

Emig, J. (1977). Writing as a mode of learning. College Composition and Communication, 28, 122-128.

Ganguli, A. B. (1989). Integrating writing in developmental mathematics. College Teaching, 37(4), 140-142.

Griffin, C. W. (1983). Using writing to teach many disciplines. Improving College and University Teaching, 31(3), 121-128.

Hirsch, L. R., & King, B. (1983). The relative effectiveness of writing assignments in an elementary algebra course for college students. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Quebec, Canada.

Irmscher, W. (1979). Writing as a way of learning and developing. College Composition and Communication, 30, 240-244.

Johnson, M. L. (1983). Writing in the mathematics class: A valuable tool for learning. Mathematics Teacher, 76, 117-119.

Kenyon, R. W. (1988). Writing is problem solving (Report No. SE 050083). Amherst, MA: University of Massachusetts, Scientific Reasoning Research Institute. (Eric Document Reproduction Service No. ED 300 247)

Miller, L. D., & England, D. A. (1989). Writing to learn algebra. School Science and Mathematics, 89, 299-312.

Nahrgang, C., & Peterson, B. (1986). Using writing to learn mathematics. Mathematics Teacher, 79, 461-465.

National Council for the Teachers of Mathematics. (1980). Priorities in school mathematics. Reston, VA: Author.

National Council for the Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: Author.

Odell, L. (1980). The process of writing and the process of learning. College Composition and Communication, 31, 42-51.

Powell, A. B., & Lopez J. A. (1989). Writing as a vehicle to learn mathematics: A case study. In P. Connolly & T. Vilardi (Eds.), Writing to learn mathematics and science (pp. 157-177). New York: Teachers College Press.

Resnick, L. B. (1987). Education and learning to think. Washington, DC: National Academy Press.

Schoenfeld, A. H. (1980). Teaching problem solving skills. American Mathematics Monthly, 87, 794-805.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem-solving skills. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 345-395). New York: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 334-371). New York: Macmillan.

Silver, E. A. (1982). Thinking about problem solving: Toward an understanding of metacognitive aspects of mathematical problem solving. Paper presented at the Conference on Thinking, Saba, Figi.

Silver, E. A., Branca, N. A., & Adams, V. M. (1980). Metacognition: The missing link in problem solving? In R. Karplus (Ed.), Proceedings of the Fourth International Conference for the Psychology of Mathematics Education (pp. 213-222). Berkley, CA: University of California.

Vygotsky, L. S. (1962). Thought and language. Cambridge, MA: MIT Press.